Simultaneous game of 3 players

We have to distribute 50 among 3 players. Each player can choose $x_i \in [0, K]$ where x_i is an integer. Additionally, if the sum of what each player chooses exceeds 50, then the 50 pesos go to player 3 and the other players get 0. But if the sum is less than or equal to 50, each player gets what they chose. That is, the payments would be:

$$(0; 0; 50) \text{ if } x_1 + x_2 + x_3 > 50$$

$$(x_1, x_2, x_3) \text{ if } x_1 + x_2 + x_3 \leq 50$$

1. If $K = 40$, determine if the following cases are NE, and if there are incentives to deviate, indicate which player has the incentive to deviate and towards what strategy.

    a) $(40, 40, 40)$
    b) $(0, 0, 40)$
    c) $(20, 20, 40)$
    d) $(25, 25, 25)$

2. Find all NE if $K = 60$.

Answers

1. *a*) It is a Nash equilibrium because even if player 1 or 2 changes their choice, the sum will still be greater than 50 and therefore they will have a payoff of 0. On the other hand, player 3 has the maximum payoff (50) and therefore has no incentive to deviate.

   b) It is not a Nash equilibrium because both player 1 and player 2 can choose a number between 1 and 10 and receive a payoff greater than 0 since the sum would not exceed 50.

   c) It is a Nash equilibrium, just like in the first case, player 1 and player 2 cannot receive a payoff greater than 0 no matter what they do, and player 3 has no incentive to lower their choice since they are obtaining the maximum payoff.

   d) It is a Nash equilibrium, even if player 1 or player 2 chooses a lower number, the payoff they receive is still 0. Meanwhile, player 3 cannot obtain a higher payoff.

2. If $K = 60$, then the Nash equilibria are as follows:

   - (x_1, x_2, x_3) where $x_3 \in [50, 60]$ and $x_1, x_2 \in [0, 60]$. For example, $(0, 0, 50)$, $(10, 4, 60)$, $(2, 1, 55)$.
   - (x_1, x_2, x_3) where $x_3 \in [0, 49]$, $x_1 + x_3 > 50$, and $x_2 + x_3 > 50$. For example, $(20, 15, 45)$, $(2, 2, 49)$, $(10, 5, 46)$.
   - (x_1, x_2, x_3) where $x_3 \in [0, 49]$, $x_1 + x_2 + x_3 > 50$, $x_1 + x_3 = 50$, $x_2 + x_3 = 50$. For example, $(1, 1, 49)$, $(4, 4, 46)$.